

Price competition and managerial delegation under partial cooperation

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Abstract: This paper analyzes a three-stage Bertrand duopoly with partial cooperation and managerial delegation. In the first stage, each owner decides whether to hire a manager. In the second stage, owners who hire managers select their managers' incentive parameters. In the third stage, managers—or owners if no delegation occurs—simultaneously and independently set the firms' prices. The equilibrium is derived using backward induction under a subgame-perfect equilibrium. As a result, the paper shows that managerial delegation does not increase payoffs for either firm. This finding contrasts sharply with the Cournot case, where delegation is profitable, and highlights that the strategic value of delegation depends critically on the mode of competition.

Keywords: Bertrand model; Managerial delegation; Partially cooperating firm; Subgame perfection

I. INTRODUCTION

The modern theory of managerial delegation originates with the seminal contributions of Fershtman and Judd (1987) and Sklivas (1987), who showed that owners can strategically influence market outcomes by designing incentive schemes that induce managers to behave more aggressively or more softly than pure profit maximizers. This foundational insight generated a large literature examining how alternative managerial objectives shape competition. Barros (1995) and Basu (1995) demonstrated that incentive schemes can alter equilibrium behavior in mixed and Stackelberg markets, while Kräkel (2002) analyzed strategic incentives in contests. Fumas (1992) and Miller and Pazgal (2001, 2002, 2005) established that relative performance evaluation can serve as a commitment device, often changing the equivalence between price and quantity competition and influencing firms' strategic aggressiveness. Behavioral motivations such as fairness preferences (Fehr and Schmidt, 1999) further justify managerial objectives that deviate from strict profit maximization.

A related strand of research studies cooperation and collusion in oligopoly. Cyert and DeGroot (1973) introduced learning and cooperative behavior in duopoly settings, while Escrihuela-Villar (2015) clarified the formal equivalence between conjectural variations and cooperation coefficients. Related work—including Lambertini and Trombetta (2002), Matsumura and Matsushima (2012), Pal (2010), and Spagnolo (2005)—showed that managerial incentives can either facilitate or hinder collusion depending on the structure of contracts and the competitive environment. Hamada (2021) revisited cooperative delegation by incorporating

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cost-based managerial objectives, demonstrating how such incentives affect the feasibility of cooperative outcomes. Fanti, Gori, and Sodini (2012) added a dynamic perspective, showing that relative-profit delegation can generate nonlinear dynamics, including cycles and chaotic behavior.

A substantial body of research examines mixed oligopolies, where public firms, labor-managed firms, or foreign competitors interact with private firms. Bárcena-Ruiz (2009), Fernández-Ruiz (2009), Ohnishi (2018a, 2018b, 2020, 2021), Uttara (2016), and White (2001) analyze the endogenous decision to hire managers and the optimal design of managerial incentives in markets with heterogeneous ownership structures. These studies show that privatization, public objectives, and labor-management considerations significantly influence the strategic role of delegation. Earlier work by Fershtman (1990) and Stewart (1992) similarly emphasized the interaction between ownership form and market structure, demonstrating that ownership status shapes firms' incentives to delegate and the resulting competitive behavior. Macho-Stadler and Verdier (1991) added the role of cross-ownership, showing how ownership links modify managerial incentives and strategic interactions.

A rapidly expanding literature focuses on common ownership, in which institutional investors hold shares in multiple competing firms. Theoretical contributions—including Chen, Matsumura, and Zeng (2024), Hirose and Matsumura (2022), and Sato and Matsumura (2020)—show that common ownership can reduce competitive pressure, alter firms' strategic incentives, and affect welfare in settings ranging from free-entry markets to vertically related industries and environmental CSR decisions.

Another important line of research examines partial cooperation and threshold behavior in oligopoly. Matsumoto, Merlone, and Szidarovszky (2010) analyzed dynamic oligopoly with partial cooperation and antitrust thresholds, while Szidarovszky (2008) studied capacity constraints and threshold effects. Ohnishi (2023) extended partial-cooperation models to both substitute and complementary goods, and Novo-Peteiro (2026) revisited cooperative delegation by integrating partial cooperation into managerial-delegation frameworks. These models complement the conjectural-variation and cooperative-delegation literatures by allowing cooperation to vary continuously rather than discretely.

Finally, recent work by Ohnishi (2026) integrates managerial delegation into a Cournot duopoly with partially cooperating firms, showing how cooperative weights influence the incentives to delegate and the structure of optimal managerial contracts. This contribution highlights the importance of jointly analyzing delegation and cooperation—two mechanisms that have traditionally been studied separately.

We extend Ohnishi (2026) by analyzing a three-stage Bertrand duopoly model with partial cooperation and managerial delegation. We consider a price-setting duopoly in which two partially cooperating firms coexist. We show that managerial delegation does not increase payoffs for either firm, and we find that this outcome contrasts sharply with that of the Cournot model in Ohnishi (2026).

The remainder of this paper proceeds as follows. In Section II, we describe the model. Section III examines three fixed-timing games: neither firm hires a manager, only one firm hires a manager, and both firms hire managers. Section IV presents the model's equilibrium. Finally, Section V concludes the paper.

II. MODEL

We consider a model with two partially cooperating firms, firm 1 and firm 2, that produce heterogeneous products. Throughout this paper, subscripts 1 and 2 represent firm 1 and firm 2, respectively. In addition, when i and j are used to represent firms in an expression, they should be understood to refer to 1 and 2 with $i \neq j$. There is no possibility of entry or exit. Each firm can hire

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one manager to make its production decisions. The inverse demand function is given by $p_i = 10 - q_i - \gamma q_j$, where p_i denotes the price of product i , q_i and q_j are respectively outputs of firm i and firm j , and γ is an index of product differentiation. We consider that firms compete by setting prices, and accordingly, the demand function faced by firm i is $q_i = \frac{a}{1+\gamma} - \frac{1}{1-\gamma^2} p_i + \frac{\gamma}{1-\gamma^2} p_j$. We assume that $\gamma = 0.5$. Production technologies of both firms are assumed to be identical. Firm i 's profit function is given by $\pi_i = (p_i - c)q_i$, where c denotes the marginal cost of production. We assume that $c = 1$.

Therefore, firm i 's objective function is given by

$$\Pi_i = \pi_i + \lambda \pi_j, \quad (1)$$

where λ denotes the degree of common ownership. We assume that $\lambda = 0.25$. Each firm seeks to maximize (1).

Firm i has the option to hire a manager to determine its price. The manager of firm i seeks to maximize a function of Π_i and R_i :

$$M_i = \theta_i \Pi_i + (1 - \theta_i) R_i, \quad (2)$$

where $R_i = p_i q_i$ denotes revenue and $\theta_i \in [0, 1]$ is the incentive parameter chosen by the owner of firm i . Using symmetry, we assume $\theta_1 = \theta_2 = \theta$.

Following Fernández-Ruiz (2009) and White (2001), we consider a three-stage game. In stage one, each owner decides whether to hire a manager. In stage two, owners who hire managers select their managers' incentive parameters. In stage three, managers — or owners if no delegation occurs — simultaneously and independently set the firms' prices. The equilibrium is derived using backward induction under a subgame-perfect equilibrium.

III. FIXED-TIMING GAMES

In this section, we examine three fixed-timing games:

- (i) Neither firm hires a manager;
- (ii) Only firm 1 hires a manager;
- (iii) Both firms hire managers.

We discuss each case in turn.

(i) Neither firm hires a manager

When neither firm hires a manager, the simultaneous maximization of these owners' objective functions yields the following Bertrand reaction function at stage three:

$$p_i(p_j) = \frac{47 + 5p_j}{16}.$$

Firm i therefore maximizes its objective function given by (1). Solving the reaction functions simultaneously gives the equilibrium prices:

$$p_i^N = 4 \frac{3}{11},$$

where the superscript N denotes that neither firm hires a manager. These price choices imply:

$$q_i^N = 3 \frac{9}{11}, \quad \pi_i^N = 12 \frac{60}{121}, \quad \Pi_i^N = 15 \frac{60}{121}.$$

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(ii) Only firm 1 hires a manager

In stage three, the manager of firm 1 maximizes the objective function given by (2), while firm 2 maximizes (1). Their reaction functions are:

$$p_1(p_2) = \frac{40 + 7\theta_1 + (4 + \theta_1)p_2}{16},$$

$$p_2(p_1) = \frac{47 + 5p_1}{16}.$$

Solving these yields the equilibrium price functions:

$$p_1 = \frac{3(276 + 53\theta_1)}{236 - 5\theta_1}, \quad p_2 = \frac{7(136 + 5\theta_1)}{236 - 5\theta_1}.$$

In stage two, firm 1 chooses the incentive parameter θ_1 to maximize its own objective function:

$$\Pi_1 = \frac{14(57,628 + 4,489\theta_1 - 2,582\theta_1^2)}{(236 - 5\theta_1)^2}.$$

Accordingly, we derive the first-order condition:

$$\frac{\partial \Pi_1}{\partial \theta_1} = 41,859,065\theta_1^2 - 2,033,468,808\theta_1 + 2,702,149,968 = 0.$$

This leads to the following choice of incentive parameter:

$$\theta_1 = \theta^O = 1,$$

where the superscript O denotes that only firm 1 hires a manager. This case results in the same equilibrium prices, outputs, and payoffs as in (i).

(iii) Both firms hire managers

When both firms hire managers, the simultaneous maximization of their objective functions yields the following Bertrand reaction functions in stage three:

$$p_i(p_j) = \frac{40 + 7\theta_i + (4 + \theta_i)p_j}{16}.$$

Solving these reaction functions simultaneously gives the equilibrium price functions:

$$p_i = \frac{800 + 152\theta_i + 28\theta_j + 7\theta_i\theta_j}{240 - 4\theta_i - 4\theta_j - \theta_i\theta_j}.$$

In stage two, the owner of firm i choose θ_i to maximize (1):

$$\Pi_i = \frac{2(1,120,000 + 93,280\theta_i + 100,720\theta_j + 2,380\theta_i\theta_j - 48,848\theta_i^2 - 10,532\theta_j^2 - 5,204\theta_i^2\theta_j - 1,391\theta_i\theta_j^2 - 170\theta_i^2\theta_j^2)}{3(240 - 4\theta_i - 4\theta_j - \theta_i\theta_j)^2}.$$

Accordingly, we derive the first-order condition:

$$\frac{\partial \Pi_i}{\partial \theta_i} = \theta_i\theta_j^3 + 500\theta_j^3 + 1,392\theta_i\theta_j^2 + 7,296\theta_j^2 + 64,656\theta_i\theta_j + 744,320\theta_i - 104,640\theta_j - 1,011,200 = 0,$$

which this can be rewritten as:

$$\theta_i(\theta_j) = -\frac{4(125\theta_j^2 - 676\theta_j - 12,640)}{\theta_j^2 + 1,372\theta_j + 37,216}.$$

Using firm symmetry, we obtain the equilibrium incentive parameter:

$$\theta_i = \theta_j = \theta^B = 1,$$

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where the superscript B indicates that both firms hire managers. This case also yields the same equilibrium values as in (i).

IV. EQUILIBRIUM

In this section, we present the equilibrium of the model introduced in Section II. As shown in the previous section, $\Pi_i^N = \Pi_i^O = \Pi_i^B$.

The main result of this paper is stated in the following proposition.

PROPOSITION 1: *In the three-stage Bertrand duopoly with partial cooperation, managerial delegation does not increase payoffs for either firm.*

Consider the three subgames analyzed in Section III. When neither firm hires a manager, the simultaneous maximization of the owners' objective functions yields the equilibrium prices p_i^N , equilibrium outputs q_i^N , and equilibrium payoffs Π_i^N .

When only firm 1 hires a manager, firm 1 chooses its incentive parameter θ in the second stage to maximize its own objective function. Solving the first-order condition shows that the optimal incentive parameter is $\theta^O = 1$. Substituting this value into equation (2) yields equilibrium prices, equilibrium outputs, and equilibrium payoffs identical to those in the benchmark case without delegation.

When both firms hire managers, each owner chooses its incentive parameter to maximize its own objective function. The first-order conditions imply that the unique symmetric solution is $\theta^B = 1$. Substituting this value into equation (2) again produces the same equilibrium prices, equilibrium outputs, and equilibrium payoffs as in the case without delegation.

Hence, in all three organizational structures—no delegation, unilateral delegation, and bilateral delegation—the equilibrium payoffs satisfy $\Pi_1^N = \Pi_1^O = \Pi_1^B$ and $\Pi_2^N = \Pi_2^O = \Pi_2^B$. Hence, delegation does not increase payoffs for either firm.

The result stands in sharp contrast to the Cournot model analyzed in Ohnishi (2026), where both firms strictly prefer to hire managers in equilibrium. Under quantity competition, managerial delegation alters the firms' strategic aggressiveness and leads to higher payoffs.

In the present Bertrand setting, however, delegation does not change equilibrium prices and equilibrium payoffs, and thus provides no strategic benefit. This comparison highlights a fundamental asymmetry between price and quantity competition: delegation is profitable under Cournot competition but strategically irrelevant under Bertrand competition.

The difference underscores the importance of market structure in determining whether managerial incentives serve as an effective commitment device.

V. CONCLUSION

We have examined a three-stage Bertrand duopoly model with partial cooperation and managerial delegation. We have shown that managerial delegation does not increase payoffs for either firm. This result contrasts sharply with the Cournot case, where delegation is profitable. Our analysis highlights that the strategic value of delegation depends critically on the mode of competition. Future research may extend this framework to alternative market structures or different forms of cooperation.

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