

Managerial Delegation in a Cournot Duopoly with Partially Cooperating Firms

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Abstract: This study investigates a quantity-setting duopoly model in which two partially cooperating firms coexist. The following three-stage game is considered. In the first stage, each owner decides whether to hire a manager. In the second stage, the owners who hired managers select incentive parameters for them. In the third stage, the managers or, in their absence, the owners simultaneously and independently choose the firms' outputs. The study adopts subgame perfection as an equilibrium concept and solves the game by backward induction. The study demonstrates that, in the subgame-perfect equilibrium, both firms hire managers.

Keywords: Managerial delegation; Mixed duopoly; Cournot model; Partially cooperating firm

JEL classification: C72, D21, D43, L13

I. INTRODUCTION

The theoretical foundation of the managerial delegation literature originates with the seminal contributions of Fershtman and Judd (1987) and Sklivas (1987), who demonstrated that owners can strategically alter market outcomes by designing incentive schemes that induce managers to behave more aggressively or more softly than pure profit maximizers. Subsequent work—including Barros (1995), Basu (1995), Fumas (1992), Kräkel (2002), and the influential series by Miller and Pazgal (2001, 2002, 2005)—extended this framework to price and quantity competition, international markets, and contests, showing that relative performance evaluation and sales-based incentives can fundamentally reshape equilibrium conduct. Behavioral motivations, such as fairness preferences (Fehr and Schmidt 1999), further explain why managerial objectives may systematically deviate from pure profit maximization.

A parallel strand of research examines cooperation, conjectural variations, and the stability of collusive behavior. Early work by Cyert and DeGroot (1973) introduced learning and cooperation in duopoly settings, while Escrihuela-Villar (2015) clarified the formal equivalence between conjectural variation equilibria and cooperation coefficients. Studies such as Lambertini and Trombetta (2002), Macho-Stadler and Verdier (1991), Matsumoto et al. (2010), Matsumura and Matsushima (2012), Pal (2010), and Spagnolo (2005) show that managerial delegation can either facilitate or hinder collusion depending on the structure of incentives. More recent contributions, including Fanti et al. (2012) and Hamada (2021), highlight how alternative managerial objectives—such as cost-based incentives or relative profit motives—can generate complex dynamics and alter the feasibility of cooperative outcomes.

Another important branch of the literature examines mixed oligopolies, in which public firms, labor-managed firms, or foreign competitors interact with private firms. Research by Bárcena-Ruiz (2009), Fernández-Ruiz (2009), Outtara (2016), White (2001), and

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a series of papers by Ohnishi (2018a, 2018b, 2020, 2021) analyzes the endogenous decision to hire managers and the optimal design of managerial incentives in markets with heterogeneous ownership structures. These studies show that privatization, public firm objectives, and labor-management considerations significantly influence the strategic role of delegation. This line of work builds on earlier insights by Fershtman (1990) and Stewart (1992) regarding the interaction between ownership form and market structure.

A more recent and rapidly expanding literature focuses on common ownership, in which institutional investors hold shares in multiple competing firms. Empirical work by Backus et al. (2019) and Schmalz (2018) documents the rise of common ownership in the United States and its potential to soften competition. Theoretical contributions—including Chen et al. (2024), Hirose and Matsumura (2022), and Sato and Matsumura (2020)—show that common ownership can reduce competitive pressure, alter firms' strategic incentives, and affect welfare in settings ranging from free-entry markets to vertically related industries and environmental corporate social responsibility decisions.

Research on partial cooperation and threshold behavior—such as Matsumoto et al. (2010), Ohnishi (2023), and Szidarovszky (2008)—provides an additional framework for modeling intermediate degrees of cooperation in duopoly games, incorporating capacity constraints, antitrust thresholds, and the substitutability or complementarity of goods. These models complement the conjectural variation and cooperative delegation literatures by allowing cooperation to vary continuously rather than discretely.

Taken together, these strands of research show that ownership structure, managerial incentives, and cooperative behavior jointly shape competitive outcomes in oligopoly markets. Yet despite extensive work on delegation, cooperation, mixed oligopolies, and common ownership, the interaction between these mechanisms remains only partially understood. This gap motivates further theoretical analysis that integrates managerial delegation with cooperative behavior and modern ownership patterns to better capture the strategic realities of contemporary markets.

Macho-Stadler and Verdier (1991) focus on the strategic use of managerial incentives in the context of cross-ownership structures within oligopolies. Macho-Stadler and Verdier examine how cross-ownership influences the design of managerial contracts and competitive behavior, emphasizing the role of ownership ties in shaping strategic interactions between firms. Fanti et al. (2012) investigate the nonlinear dynamics of a Cournot duopoly in which firms delegate decision-making to managers rewarded according to relative profit incentives. Fanti et al. reveal that such delegation can generate complex dynamic behavior—including cycles and chaos—depending on the structure of incentive schemes. Similarly, Novo-Peteiro (2026) revisits the relationship between cooperation and managerial delegation in duopoly markets. Novo-Peteiro demonstrates that cooperative managerial incentives can replicate Bertrand outcomes under specific conditions, challenging the traditional view that cooperation and delegation reduce welfare. Novo-Peteiro highlights how cooperative behavior influences managerial delegation and the design of incentive contracts, offering insights into how strategic cooperation can improve market performance.

Building on these contributions, this paper examines managerial delegation in a Cournot duopoly with partially cooperating firms, where each owner places positive weight on the rival's profit. By embedding delegation into a partially cooperative environment, we analyze how cooperative weights influence the incentives to delegate, the structure of optimal managerial contracts, and the resulting market outcomes. We study a three-stage Cournot duopoly model with two partially cooperating firms and show that both firms optimally hire managers in the subgame-perfect equilibrium, providing new insights into the strategic role of delegation in partially cooperative duopolies.

The rest of this paper is organized as follows. In Section II, we describe the model. Section III examines the following three subgames: neither firm hires a manager, only firm 1 hires a manager, and both firms hire managers. Section IV presents the model's equilibrium. Finally, Section V concludes the paper.

II. MODEL

Let us consider a model with two partially cooperating firms: firm 1 and firm 2. Each firm can hire one manager to make its production decisions. There is no possibility of entry or exit. The duopolists produce perfectly substitutable commodities. The

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market price is determined by the inverse demand function, $P(Q) = 10 - Q$, where $Q = q_1 + q_2$ denotes the aggregate quantity.

Firm i 's profit function is given by

$$\pi_i = P(Q)q_i - mq_i, \quad (i = 1, 2), \quad (1)$$

where m denotes the constant marginal cost. We assume that $m = 1$.

Firm i 's objective function is given by

$$\Pi_i = P(Q)q_i - q_i + \lambda [P(Q)q_j - q_j], \quad (i, j = 1, 2; i \neq j), \quad (2)$$

where λ denotes the degree of common ownership. We assume that $\lambda = 1/2$. Each firm aims to maximize (2).

Firm i can hire a manager to make its production decisions. The manager of firm i seeks to maximize a function of Π_i and R_i :

$$M_i = \theta_i \Pi_i + (1 - \theta_i) R_i, \quad (3)$$

where $R_i = P(Q)q_i$ denotes the revenue and $\theta_i \in [0, 1]$ denotes the incentive parameter chosen by the owner of firm i . In this study, using the symmetry of firms, we assume that $\theta_1 = \theta_2 = \theta$.

We follow White (2001) and Fernández-Ruiz (2009) and consider the following three-stage game. In the first stage, each owner decides whether to hire a manager. In the second stage, the owners who hired managers select incentive parameters for them. In the third stage, the managers or, in their absence, the owners simultaneously and independently choose the firms' outputs. We adopt subgame perfection as an equilibrium concept and solve the game by backward induction.

III. ANALYSIS OF THE DIFFERENT HIRING DECISIONS

In this section, we examine the following three cases:

- (i) Neither firm hires a manager.
- (ii) Only firm 1 hires a manager.
- (iii) Both firms hire managers.

We discuss these cases in order.

(i) Neither firm hires a manager

When neither firm hires a manager, the simultaneous maximization of these managers' objective functions leads to the following Cournot reaction function at the third stage:

$$q_i^n = \frac{18 - 3q_j}{4}, \quad (i, j = 1, 2; i \neq j),$$

where the superscript n denotes that neither firm hires a manager. Therefore, firm i will maximize its own objective function given by (2). Solving these reaction functions simultaneously allows the following equilibrium outputs:

$$q_i^n = 2\frac{4}{7}, \quad (i = 1, 2).$$

These output choices imply:

$$P^n = 4\frac{6}{7}, \quad \pi_i^n = 9\frac{45}{49}, \quad \Pi_i^n = 14\frac{43}{49}.$$

(ii) Only firm 1 hires a manager

In the third stage, the manager of firm 1 maximizes the objective function given by (3), while firm 2 maximizes (2). Reaction functions of firm 1 and firm 2 are respectively given by

$$q_1^o = \frac{18\theta - (2 - \theta)q_2}{4},$$

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$$q_2^o = \frac{18 - 3q_1}{4},$$

where the superscript o denotes that only firm 1 hires a manager.

These lead to the following output functions:

$$q_1^o = \frac{18(3\theta - 2)}{10 - 3\theta}, \quad q_2^o = \frac{18(4 - 3\theta)}{10 - 3\theta}.$$

In the second stage, firm 1 chooses the incentive parameter θ to maximize its own objective function. In the second stage, the owner of each firm chooses θ to maximize (2). Simultaneous maximization of these objective functions leads to the following choice of incentive parameters:

$$\theta^o = 1.$$

This results in:

$$q_i^o = 2\frac{4}{7}, \quad P^o = 4\frac{6}{7},$$

$$\pi_i^o = 9\frac{45}{49}, \quad \Pi_i^o = 14\frac{43}{49}, \quad (i = 1, 2).$$

Note that the equilibrium values of this subgame are the same as those of (i).

(iii) *Both firms hire managers*

When both firms hire managers, the simultaneous maximization of these managers' objective functions leads to the following Cournot reaction function at the third stage:

$$q_i^b = \frac{18\theta - (2 - \theta)q_j}{4}, \quad (i, j = 1, 2; i \neq j),$$

where the superscript b indicates that both firms hire managers.

Solving these managers' reaction functions simultaneously allows the following equilibrium outputs:

$$q_i^b = \frac{18\theta}{6 + \theta}, \quad (i = 1, 2).$$

In the second stage, each firm chooses θ to maximize (2). Simultaneous maximization of these objective functions leads to the following choice of incentive parameters:

$$\theta^b = \frac{6}{7}.$$

This results in:

$$q_i^b = 2\frac{1}{4}, \quad P^b = 5\frac{1}{2},$$

$$\pi_i^b = 10\frac{1}{8}, \quad \Pi_i^b = 15\frac{3}{16}.$$

IV. EQUILIBRIUM

In this section, we present the equilibrium of the model introduced in Section II. From the discussions in the previous section, we see that $\Pi_i^n = \Pi_i^o < \Pi_i^b$.

The main result of this study is described by the following proposition:

PROPOSITION 1: *In the subgame perfect equilibrium, both firms hire managers.*

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This result highlights the strategic advantage of managerial delegation in a Cournot duopoly with partially cooperating firms. By hiring managers, both firms can optimize their production decisions and achieve higher objective function values than in scenarios where either firm or neither firm employs managers. The equilibrium outcome underscores the role of managerial incentives in shaping competitive dynamics and suggests that delegation is a robust strategy even in environments characterized by partial cooperation. Furthermore, this finding contributes to the broader literature on oligopoly theory by integrating the concepts of managerial delegation and partial cooperation, offering new insights into how firms can strategically align their objectives to improve market performance.

V. CONCLUSION

We have studied a three-stage Cournot duopoly model in which two partially cooperating firms coexist. Each firm first decides whether to hire a manager. We have demonstrated that, in the subgame-perfect equilibrium, both firms hire managers. As a result, we find that introducing managerial delegation into the analysis of Cournot duopoly competition is profitable for partially cooperating firms. Future research could explore the implications of this result in more complex market structures or under different forms of cooperation.

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