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Impact of Graphs and Network in Minimizing Project and Product Cost

Petrina (Pei-Chun) C. Feng
Dept. of Administrative Management, Central Police University, Taoyuan, Taiwan

Abstract
The world has many natural systems that are so complex to be understood easily. This creates a need to have simple principles or systems that capture the complexity of the world. The simple systems make it easier for many people to understand the world by representing the complex world in a more straightforward way (Stefan, 2003). Many objects and projects are seen to be a network of processes or substances. Graphs and networks have been used widely in different projects for different reasons by project managers mostly. There are techniques such as critical path analysis that make use of graphs and networks and are applied by project managers and all the staff involved in projects. These methods are used to ensure smooth planning and control of projects. However, the techniques have to be applied correctly to achieve the desired objective. This paper looks at the impact of graphs and networks in minimizing the costs of a project or product. From this research, it can be inferred that the techniques such as critical path method, that make use of graphs and networks, play a significant role in determining and hence reducing the product cost. This is done by making the right decisions regarding the resources and time most appropriate for a project. The paper shows clearly how these techniques are applied in a project to determine project duration and hence minimize the cost.

Keywords: Graphs, Minimizing cost, Network.

I. INTRODUCTION

The world has many natural systems that are so complex to be understood easily. This creates a need to have simple principles or systems that capture the complexity of the world. The simple systems make it easier for many people to understand the world by representing the complex world in a more straightforward way(Stefan, 2003). One of these simple principles is networks.

The world is believed to be an interconnection of many things that eventually form a network. Some of the examples of these networks in the world include the interconnection of molecules that are contained in a living cell, interconnection of computers which share information all over the world, the nerve cells that are found in the brains of each human being, the social networks such as twitter where many people all over the world are interconnected (Newman,2010).

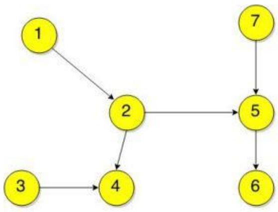
Graphs, on the other hand, may be regarded as tools which are used to study the interrelationships between objects. Graphs are another simple system that is used to represent the complexities in the word. There are several types of graphs and each type of graph has its use. The theory used in graphs represents both local and global characteristics of an object. In such a case when numerical characteristics are associated with points or lines in a graph, we may end up generating a model that may be a mathematical, physical or social nature (Saaty & Busacker, 1965). Graphs are majorly used to study networks. In a business set up, project networks make use of graphs where we have nodes and edges connecting the nodes. Graphs, in this case, can be used to determine the project with minimal costs.

This research paper discusses graphs and networks and how they can be used to determine a project that has minimal cost. An organization can use this to decide on what projects to invest in and determine the approximate amount that a project is likely to cost.

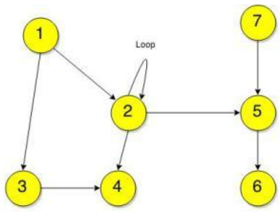
II. GRAPH THEORY

A graph can be defined as a countable number of edges and vertices. It can be represented mathematically as G(V,E) where V is a set of vertices also referred to as nodes and E is a set of edges that interconnect the nodes or are used to show associations between these nodes. The edges may be representing things such as lengths, weights, costs or heights. There are two types of graphs where the edges show no direction. Trees in mathematics can be referred to as the undirected graphs. A rooted tree may, however, be directed if the edges point towards the root or away from the root.

On the other hand, the directional graphs refer to the kind of graphs where the edges are directional (Bollobas, 1979). The other types of graphs are as shown below.

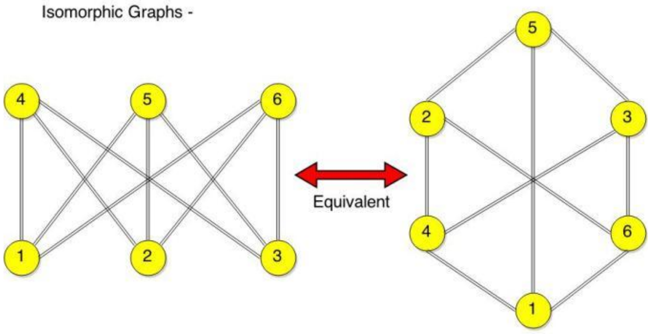


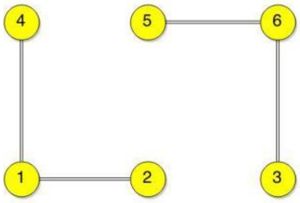
Simple Graph



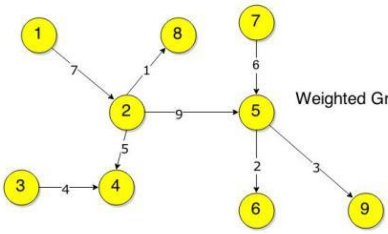
Non - Simple Graph

Isomorphic Graphs -





Disconnected Graph



Weighted Graph

In graph, therefore, can be thought of as a collection of vertices that are joined together by edges. The number of edges gives the size of the graph. The number of vertices represents the order of the graph G (Stefan, 2003). Graphs are mostly used to study networks and different kinds of networks have different kinds of graphs that are most suitable for them. The graphical theory has a wide range of applications which include biochemistry, mathematics, computer science and electrical engineering among others.

In real life, the nodes are used to represent projects or activities, and the edges are used to describe the process costs, lengths or weights. From the network of the projects or project activities in a graph, the minimum costs, length or weight represented by the edges can be determined. This can be done by the used of short path problem, which refers to finding a path between two vertices whose sum of the edges or weights has been minimized. There are several algorithms for solving this problem which includes, Dijkstra's algorithm, Bellman-Ford algorithm, A search algorithm, Floyd-Warshall algorithm, Johnson's algorithm, and Viterbi algorithm. Each of these algorithms has a formula for solving the short path problem (MacCrimmon, 1987). An adjacency matrix which is a two-dimensional matrix can be used to generate or represent a graph. A graph can also be represented using an adjacency list which is an array of pointers that are used to describe vertices that are adjacent to Vi (Lun et, al. 2004). For the above graph, the adjacency matrix would be as below.

to	Vertex 1	Vertex 2	Vertex 3	Vertex 4	Vertex 5	Vertex 6	Vertex 7	Vertex 8	Vertex 9
Vertex 1	0	7	0	0	0	0	0	0	0
Vertex 2	0	0	0	5	9	0	0	1	0
Vertex 3	0	0	0	4	0	0	0	0	0
Vertex 4	0	0	0	0	0	0	0	0	0
Vertex 5	0	0	0	0	0	2	0	0	3
Vertex 6	0	0	0	0	0	0	0	0	0
Vertex 7	0	0	0	0	6	0	0	0	0
Vertex 8	0	0	0	0	0	0	0	0	0
Vertex 9	0	0	0	0	0	0	0	0	0

III. DISCUSSION

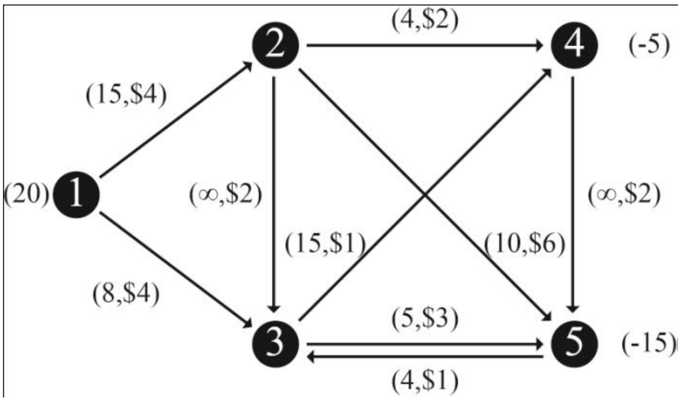
This research paper aims at investigating the role of the graphs and the network to decide on the best projects or products to invest in at the minimum cost. Project networks make use of node where different related nodes are interrelated through the arcs. Nodes are the activities while the arcs (arrows or edges) represent the prioritized relationships (Gabriel, 2009). Nodes are also responsible for receiving, processing and replicating the packets to other nodes through a given direction (Lun & Medard, 2014).

Several ways can be used to achieve at minimum cost in a network. These methods include the Critical Path Method (CPM). For minimum cost, there is a need to identify the critical path within the network by use of appropriate graphing. This is achieved through identification and action on the instances that would cause a delay in the entire project when they delay. The critical path saves the time costs through ensuring that the task is accomplished in the least time (James, 1959).

Another viable way to achieve this is through solving network flow problem where restricted algebraic operations are performed at the nodes allowing it to process the received packets and enable concurrent transmission and replications (Koetter, & Medard2003). Network decoding decouples the problems enabling the project to be cost-effective (Lun et, al. 2004).

IV. THE GENERAL NETWORK FLOW PROBLEM SOLUTION

This kind of problem arises from the fact that products are manufactured from a plant and are expected to reach the destination which is the consumer. There are however intermediaries such as warehouses and distribution centers that a product goes through before it gets to the consumer. This, therefore, creates a network where in the graph, the nodes are used to represent the sources intermediaries and destination of a product and the edges represent the network channels. In a practical scenario, the nodes will represent the product source, intermediary and destination and the edges or arcs will represent the flow capacity and the cost per unit from one node to the next. The aim of this is to determine the minimum cost flow problem. In the example below, 1 is the source node, and 4 and 5 are the destination nodes. 2 and 3 are the intermediaries or transshipment nodes. Node 1 has 20 products to ship, and node 4 and 5 require five products and fifteen products respectively, which is indicated by the negative signs. The objective is to find the minimum cost shipment method (Fondahl, 1961).



The problem is then transcribed into a formal linear program after generating the graphical representation into a tabular form using an adjacency matrix. The table for the above is as shown below;

	X12	X13	X23	X24	X25	X34	X35	X45	X53	Right hand side
Node 1	1	1								20
Node 2	-1		1	1	1					0
Node 3		-1	-1			1	1		-1	0
Node 4				-1		-1		1		-5
Node 5					-1		-1	-1	1	-15
Capacities	15	8	∞	4	10	15	5	∞	4	
Objective function	4	4	2	2	6	1	3	2	1	(Min)

The linear equations are then generated from the table above where the +1 coefficient shows the node from which the arc emanates and -1 shows the node to which the arc is an incident. Linear equations are formed and minimized from the above matrix, and hence the minimum product cost is determined.

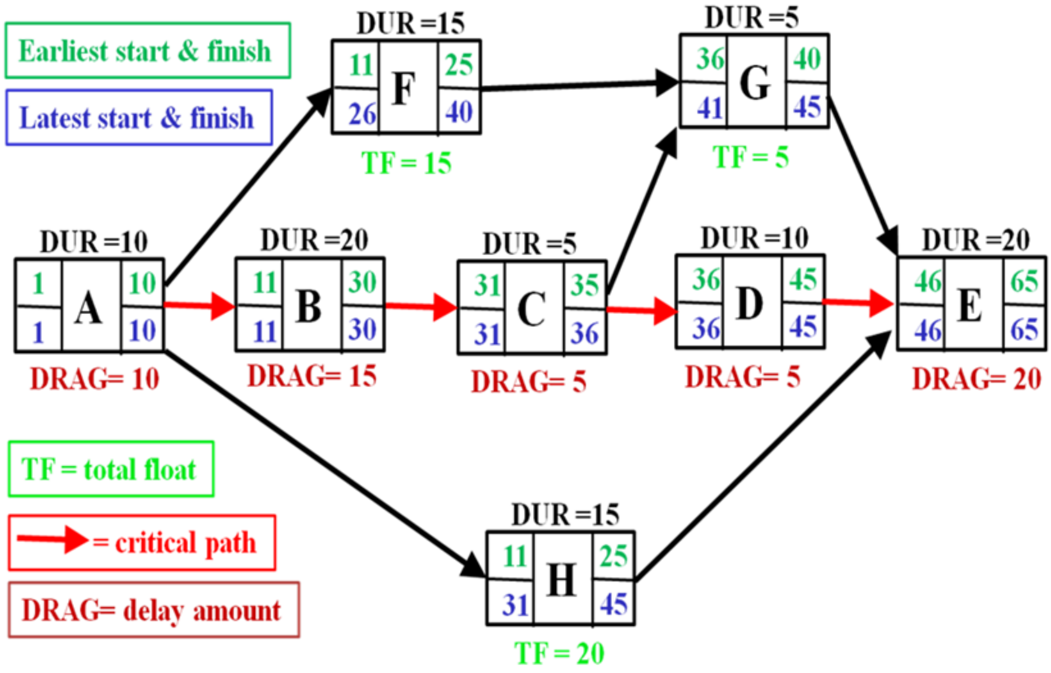
V. Critical Path Method (CPM)

This is a technique that is used for project modeling, and it was developed in the 1950s. The critical path method was developed by Morgan R. Walker, James E. Kelly Jnr who related their memories around 1989. CPM has very many applications and can be sued on different kinds of projects including research projects, construction projects, engineering projects, software development projects among others (MacCrimmon, 1987).

This method is widely applied in planning and control of big projects whose duration is known or can be determined. Before a critical path is identified, there are three things that have to be done. This includes breaking down the project into basic activities, identifying the predecessor activity that is immediate to each activity and finally does an estimation of the duration of every activity (Magallon, 2009). The activities here are the nodes, and these nodes are connected by the edges with each edge indicating the duration of each activity (Hartley, 1966). The project network will have the start node and the finish node in the network of activities. A path, in this case, can be defined as one of the routes that lead one from the start node to the finish node. From the values in a project network, we can calculate the longest and shortest path from the start node to the finish node.

CPM uses the values that are contained in a network diagram to calculate the longest path of the activities that are planned to take place during the project. CPM also calculate the earliest or latest that each activity can start without necessarily making the project to be lengthier. This process makes it easier to determine which activities are critical, i. e., the activities on the longest path and activities which can be delayed without making the project longer and these activities can be said to have the total float (Fondahl, 1961).

The critical path method helps in determining the shortest time that a project can take. Any activity which falls on the critical path cannot be delayed. If an activity on the critical path is delayed. It has a direct impact on the length of the project or completion date. A project can have several critical paths. The diagram blow shows a network of activities and the critical path.



Activities A and E are the start and finish activities respectively. Activities F, G, and H are not on the critical path, activities A, B, C, D and E fall on the critical path. If an activity in the critical path has no parallel activity, then the drag of that activity is the duration of the activity.

On the other hand, if an activity on the critical path has a parallel activity, then its drag is the duration of whichever activity is less (Hofmann, 1993).

From the computations obtained from the graph above, a project manager is able to decide on what activities can be run in parallel and also see how they can reduce the time of the activities in the critical path. The lesser the time used in a project, the more the resources required and therefore the cost of the project is expected to go up. On the other hand, if we increase the duration of project the number of resources required to complete the project such as labor will reduce and hence the cost of the project is expected to go down (James, 1959).

VI. Conclusion

Graphs and Networks are very applicable in planning and control of projects and products. This is very useful and helps in making financial and other forms of decisions regarding projects. They help in determining unnecessary costs which need not be incurred while carrying out a project's object. Graphs and networks play a vital role in minimizing project and product costs.

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